NOTATION

k_H = Henry's constant, = $f(P,T)$
k_H^0 = Henry's constant for zero solute partial pressure, = $f(T)$
$(k_H^0)_M$ = Henry's constant corresponding to the temperature of minimum solubility
P = total pressure
P_i = partial pressure of component i
R = gas constant
T = absolute temperature
T_M = temperature of minimum solubility
v_i = partial molar volume of component i
x = mole fraction of the solute gas in the liquid phase
y = mole fraction of the solute gas in the gas phase
Φ_i = fugacity coefficient of component i
^{o} = pure component
∞ = infinite dilution
1,2 = water and gaseous solute, respectively

LITERATURE CITED

Clever, H. L., and C. H. Han, "The Solubility of Gases in Water from 350–600 K," *Thermod. Aq. Syst. Ind. Appl.*, editor S. A. Newman, ACS Symp. Ser. 133, p. 513 (1980).

Crovetto, R., R. Fernández-Prini, and M. L. Japas, "Solubilities of Inert Gases and Methane in H_2O and D_2O in the Temperature Range of 300 to 600 K," *J. Chem. Phys.*, **76**, 1077 (1982).

———, "Aspectos Termodinámicos de la Solubilidad de Gases en Líquidos a Temperaturas Mayores que la de Ebullición Normal," *Anal. Asoc. Quím. Arg.*, **71**, 81 (1983).

Himmelblau, D. N., "Solubility of Inert Gases in Water. 0°C to Near the Critical Point of Water," *J. Chem. Eng. Data*, **5**, 10 (1960).

O'Connell, J. P., "Some Aspects of Henry's Constants and Unsymmetric Convention Activity Coefficients," *Phase Equil. Fluid Prop. Chem. Ind.*, Ed. T. S. Storvick and S. I. Sandler, ACS Symp. Ser. 60, p. 450 (1977).

Rigby, M., J. P. O'Connell, and J. M. Prausnitz, "Intermolecular Forces in Aqueous Vapor Mixtures," *Ind. Eng. Chem. Fund.*, **8**, 460 (1969).

Schultze, G., and J. M. Prausnitz, "Solubilities of Gases in Water at High Temperatures," *Ind. Eng. Chem. Fund.*, **20**, 175 (1981).

Wilhelm, E., R. Battino, and R. W. Wilcock, "Low-Pressure Solubility of Gases in Liquid Water," *Chem. Rev.*, **77**, 219 (1977).

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Turbulent Mixing in Tubes with Transverse Injection

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One of the simplest flows of confined turbulent mixing is a small transverse jet into a tube or pipe. This flow is also attractive as it is a common flow element in technology. A transverse jet at $x = 0$ introduces one fluid into the tube and at the same time injects vorticity from the nonconservative force on the axial flow in the tube. The circulation of the vorticity stirs the two fluids together. As time elapses, or correspondingly as the downstream distance increases, the magnitude of the concentration fluctuations declines. Although the fluctuations may never be exactly zero, after they reach some low value, say, 1 percent of the mean concentration, the two fluids are practically a homogeneous mixture. In many technical applications it is desirable that the mixing occur as rapidly as possible. Hence the rate of decay of the fluctuations is a useful measure of the mixing efficacy of the turbulence.

CONCENTRATION FLUCTUATION MEASUREMENTS

An aspirating probe, initially developed by Brown and Rebollo (1972), was used to measure the concentration fluctuations between two streams of gas, one of helium and the other nitrogen. Probe spatial resolution was approximately 0.1 cm. The tube inside diameter D was 2.3 cm. The transverse jet flowed through a sharp-edged hole, which was 0.1, 0.2, and 0.4 cm during the course of the experiments.

Typical Reynolds numbers are about 10^3 for the tube flow and 10^4 for the transverse jet. A porous metal plug of 40 μ pore size was positioned in the tube 3 diam. upstream of the transverse jet to generate a uniform nitrogen velocity in the tube.

Typical results are shown in Figure 1. The fluctuations, normalized by the mean concentration of the mixture, are plotted as a function of downstream distance normalized by tube diameter D for several values of the ratio of transverse to axial momentum J. For incompressible flow, as was the case here

$$J \equiv \frac{\rho_j V_j^2 A_j}{\rho U^2 A}$$

where ρ_j, V_j, and A_j are the jet density, velocity, and area at the nozzle exit. The corresponding tube quantities are ρ, U, and A. An orifice coefficient of 0.8 was assumed for all the jet orifices.

The fluctuations decline in Figure 1 approximately as $(x/D)^{-1}$ for almost all values of J tested. Figure 2 illustrates the dependence on J at a fixed downstream station. Except at very small and very large J, the fluctuations vary as J^{-1}. Also, the fluctuations seem to be independent of the jet diameter d, except perhaps at large J.

Although the radial position of the probe was changed within the tube, no transverse variation in c' was observed, except for the case of anemic jets where $J < 0.1$.

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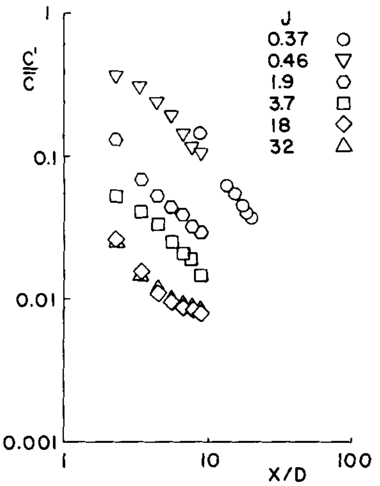


1. **Effect of momentum ratio *J* and downstream station *x/D* on the concentration fluctuations.**

THEORY

A model of the transverse jet into the tube is based on the analysis of Broadwell and Breidenthal (1984) of a free transverse jet in an infinite crossflow. They argue that the critical parameter for the jet trajectory and mixing rate is the impulse per unit length and mass of freestream fluid, I/ρ, in analogy to the lift of a wing. The spatial problem is approximated by the two-dimensional temporal problem of a line impulse. The trailing vortices generated by a point force (the lifting wing problem in the far field) depend only on the integrated force, not the size of the wing. In the same way, only the jet momentum is important, not its nozzle diameter. For incompressible flow

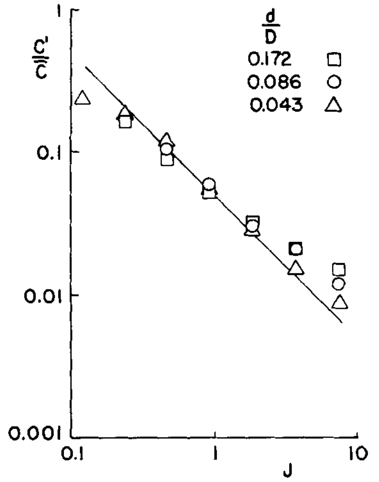
$$I/\rho = \frac{\rho_j V_j^2 A_j}{\rho U}\,.$$

The only other parameters that should enter the tube problem are the tube diameter and the Lagrangian time t. Precisely one independent dimensionless group can be formed from these three quantities, so

$$\frac{c'}{\bar{c}} = f\left(\frac{D^3}{(I/\rho)t}\right) \tag{1}$$

where f is some unknown function.

The next step is to assume that the flow and the mixing are self-similar, so that the global vorticity Ω declines as inverse time, and the concentration fluctuations decline by a constant fraction at each global rotation. Implicit is the argument of Broadwell and Breidenthal (1982) that the mixing process in free turbulence is limited by the global vorticity. Therefore



2. **Effect of jet nozzle diameter *d* and momentum ratio *J* on the mixing at *x/D* = 8.9.**

$$\frac{c'}{\bar{c}} = g e^{-\int \Omega dt} \tag{2}$$

where g is a function independent of time. If $\Omega = 1/t$, Eq. 2 implies

$$\frac{c'}{\bar{c}} = \frac{g}{t}$$

In light of Eq. 1

$$\frac{c'}{\bar{c}} = \left(\frac{K_1 D^3}{(I/\rho)t}\right)$$

with K_1 an unknown constant

Taking $t = x/U$ and A proportional to D^2, we have

$$\frac{c'}{\bar{c}} = \frac{K_2 D}{Jx} \tag{3}$$

with K_2 an undetermined constant. Equation 3 is plotted as the solid line in Figure 2 for $K_2 \doteq 0.4$. For J near unity, the agreement is satisfactory. At large J, the data diverge from the line, perhaps due to probe sensitivity to acoustic noise in the tube. At small J the model (Eq. 3) is expected to break down as the limit of classical turbulent pipe flow is approached.

It should be noted here that Eq. 2 seems correct for classical turbulent pipe flow. Hartung and Hiby (1972) found an exponential decline in concentration fluctuations as x/D is increased. Since in pipe flow both the global turbulent velocity and length scale are constrained,

$$\Omega = \frac{K_3 U}{D} = \text{constant.}$$

Equation 2 yields

$$\frac{c'}{\bar{c}} = g e^{-K_3 U t/D} = g e^{-K_3 x/D}.$$

Clearly, $g = 1$ by a judicious shift in the virtual origin, and the observations of Hartung and Hiby are matched for $K_3 \cong 0.05$.

CONCLUSION

For momentum ratios of order unity, the concentration fluctuations were found to decline inversely with momentum ratio (transverse/axial) and distance downstream of the transverse jet in a tube. These results are consistent with a simple model that assumes self-similar mixing. The model is expected to break down at very small J and large x as the classical pipe flow regime is approached. It may break down at large J as well, although the issue remains open.

ACKNOWLEDGMENT

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NOTATION

A = area

c' = RMS concentration fluctuation
$\bar{c}$ = average concentration
d = transverse jet orifice diameter
d = tube diameter
f,g = functions
I = impulse
J = ratio of transverse to axial momentum
K_i = unknown constants ($i = 1,2,3$)
t = time after injection
U = pipe velocity
V_j = jet velocity
x = distance downstream of injection point
ρ = density
Ω = global vorticity

LITERATURE CITED

Broadwell, J. E., and R. E. Breidenthal, "A Simple Model of Mixing and Chemical Reaction in a Turbulent Shear Layer," *J. Fluid Mech.*, **125,** 397 (1982).

———, "Structure and Mixing of a Transverse Jet in Incompressible Flow," *J. Fluid Mech.*, **148,** 405 (1984).

Brown, G. L., and M. R. Rebollo, "A Small, Fast-Response Probe to Measure Composition of a Binary Gas Mixture," *AIAA J.*, **10,** no. 5, 49–652 (1972).

Hartung, H. K., and J. W. Hiby, "Acceleration of Turbulent Mixing in Tubes," *Chem. Ing. Tech.*, **18,** 1051 (1972).

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Exact Solution of a Model for Diffusion in Particles and Longitudinal Dispersion in Packed Beds: Numerical Evaluation

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In an earlier paper (Rasmuson and Neretnieks, 1980) an analytical solution of a model for flow and longitudinal dispersion in beds of spherical particles, coupled with diffusion and linear sorption in the particles, was given. The solution is derived for a step inlet concentration and is in the form of an infinite integral:

$$C/C_o = \frac{1}{2} + \frac{2}{\pi}\int_o^\infty \exp[g(\delta,Pe,R,\nu,\lambda)] \times \sin[h(\delta,Pe,R,\nu,y,\lambda)]\frac{d\lambda}{\lambda} \qquad (1)$$

The entities g and h are complicated functions of the variable of integration λ and of the following dimensionless parameters:

$\delta = \frac{3D_p\epsilon_p}{b^2}\frac{z}{mV}$	bed length parameter
$Pe = \frac{zV}{D_L}$	Peclet number
$R = \frac{K}{m}$	distribution ratio
$\nu = \frac{D_p\epsilon_p}{k_f b}$	film resistance parameter
$y = \frac{2D_p\epsilon_p}{Kb^2}t$	contact time parameter

The integrand is oscillatory. Due to the very rapid oscillation of the integrand for certain parameter values, especially for longer times, a straightforward integration will fail. To overcome this difficulty a special integration method was developed. In this method the integration is performed over each half-period of the sine wave separately. The convergence of the alternating series obtained is then accelerated by repeated averaging of the partial sums.

For the case where the film resistance is negligible ($\nu = 0$), the method was utilized in a computer program for the prediction of the migration of radionuclides in fissured rock (Rasmuson and Neretnieks, 1981).

The main problem is the calculation of the zeros of

$$\sin[h(\lambda_n)] = 0 \qquad n = 1,2\ldots \qquad (2)$$

A combination of limiting expressions and Newton-Raphson's iteration is used.

The effect of ν has now been incorporated into the numerical scheme. Apart from the fact that H_1 and H_2 now are different from H_{D_1} and H_{D_2} (Rasmuson and Neretnieks, 1980) the procedure for solving Eq. 2 is modified. For $\nu > 0$ the calculations of the roots are done in two steps. First the old method is used as if $\nu = 0$. This value is then taken as the starting value for a new Newton-Raphson iteration. The convergence is then very rapid.

Some minor changes were also introduced into the code. First, the integral is calculated up to $\lambda = \lambda_{10}$ (10th root of Eq. 2). The magnitude of the integrand is then checked at the point halfway between λ_9 and λ_{10}. If the absolute value is less than 10^{-10} the integration is stopped; if not, the averaging procedure is entered. Twenty terms of the alternating series are calculated up to λ_{30}. The averaging procedure is continued until an absolute accuracy of 10^{-6} is obtained.

Some examples, showing the influence of ν, are given in Figures 1–3. Typical computing times on a CDC Cyber 170-720 are in the range of 10–35 s for 20–40 points on the concentration-time curve. The operation time for one multiplication on this computer is 4.0 μs. The running times are somewhat higher for $\nu > 0$ as compared to the case for which $\nu = 0$.

The solution was checked against the tabulated values by Rosen (1954) for $Pe = \infty$. Note that the contact time y in Rosen's notation is related to our y by